Generalized Pseudo-SU(3) Model and Pairing*

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Abstract

The pseudo-SU(3) model is extended to explicitly include the spin and proton-neutron degrees of freedom. A general formalism for evaluating matrix elements of one-body and two-body tensor operators within this framework is presented. The pairing interaction, which couples different irreducible representations of SU(3), is expressed in terms of pseudo-space tensors and a general result is given for calculating its matrix elements. The importance of pairing correlations in pseudo-SU(3) model calculations is demonstrated by examining the dependence of wavefunctions, low- energy collective excitation spectra, and moments of inertia on the strength of the pairing interaction.

1 Introduction

Over the past few years there have been a number of discoveries in nuclear structure physics (superdeformed shapes, identical bands, scissors mode, etc.) which have raised a number of interesting questions, many of which remain unanswered. In addition, one can anticipate the identification of additional new phenomena in the near future due to ongoing improvements and innovations in experimental techniques (4π detectors, radioactive beams, etc.). These experimental developments seem not to be matched by a comparable record of theoretical achievements focused on the questions that are being raised.

However, it is commonly accepted that the nuclear shell model should be able to address these issues and provide answers to these questions. The problem is that most shell-model theories are limited by the large dimensionalities of the required model spaces. For example, existing algorithms and available computing equipment still only allow for

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reasonably complete standard shell- model calculations in the mass range $A \leq 28$. And even the best of these calculations do not include a sufficient number of multi-shell configurations to adequately address the issues raised by experiments that probe details of the momentum and current distributions in nuclei. Furthermore, most of the new discoveries are for much heavier nuclei, typically in the mass range $A \geq 100$.

When faced with a situation like the one just described, it is essential to take full advantage of symmetries, those which are only approximately fulfilled as well as those that are exact. The selection rules that are associated with such symmetries generate, respectively, weakly coupled and disconnected subspaces of the full space and this in turn allows for a significant (orders of magnitude) reduction in the dimensionality of the model space. Since matrix elements of the system's Hamiltonian between these subspaces only vanish if the symmetry is exact, the question that must be addressed is whether or not using the identified approximate symmetry is physically justified.

Pseudo-spin [1, 2] is such a symmetry and it serves as the conceptual foundation for the subject of this contribution, namely, the pseudo-SU(3) model [3], and its extension, the pseudo- symplectic model [4]. The success of the pseudo-spin concept relies on the fact that the harmonic oscillator structure of the usual shell-model Hamiltonian, which is destroyed by the spin-orbit interaction, is restored under a transformation to the pseudo-space/spin picture. The idea is to express all normal-space shell-model quantities in terms of their pseudo-space equivalents by applying a normal $\leftrightarrow$ pseudo transformation to both operators and wavefunctions [5]. Since for practical purposes the harmonic oscillator structure of the Hamiltonian is restored in the pseudo-space scheme, this transformation makes it possible to invoke powerful group theoretical truncation methods so shell-model calculations can be carried out even for heavy nuclei with $A \geq 100$.

There are still questions being raised about the physical implications of the approximation one is making by assuming that the pseudo-spin symmetry is exact. However, there have been a number of publications [3, 6, 7, 8] which demonstrate in both single-particle and many- particle systems that even if it is not an exact symmetry, for practical applications pseudo-spin is a sufficiently good symmetry to be of major physical importance. The reader is referred to these publications for assurance regarding the validity of the pseudo-spin concept. Here we focus only on an algebraic many-particle shell-model theory that takes full advantage of pseudo-spin symmetry, namely, the pseudo-SU(3) model. The usefulness and predictive power of this model have been demonstrated in many application ranging from the calculation of collective excitation spectra [9] and a scissor's mode study [10] to fundamental problems likes the half-lives of $\beta\beta$-decay modes [11].

In this contribution we propose extensions of the pseudo-SU(3) model in order to widen its range of applicability and in so doing increase the capabilities of the model for describing and predicting new and interesting experimental phenomena:

- The ansatz of reference [10] is generalized by explicitly including spin degrees of freedom in a full proton- neutron formulation of the theory. This generalization makes it possible to describe properties of even-odd, odd-even, and odd-odd nuclei in addition to those of even-even ones. Within this extended approach, tensor operators are defined and expressions for calculating their matrix elements are given.

- In spite of many successful applications of the pseudo-SU(3) model, the Hamiltonian that has been used in most applications is highly schematic in nature and needs to be extended. The usual structure taken for the pseudo-SU(3) model Hamiltonian is

$$H = H_0 - \frac{\chi}{2} Q^a \cdot Q^a + R_{\rm coll} \tag{1}$$

  where H_0 is the harmonic oscillator Hamiltonian, Q^a denotes the algebraic quadrupole operator (see ref. [4]), and $R_{\rm coll}$ stands for a residual interaction that is introduced to describe properties of low-energy collective excitations. A comparison with other shell-model theories indicates the importance of introducing a short range interaction, and therefore it seems reasonable to add a pairing interaction to terms already included in Eq.(1) [12, 13].

The reason the pairing interaction has not been introduced previously is a technical matter: Basis states of the pseudo-SU(3) model are organized according to irreducible representations (irreps) of SU(3) which are labeled by (λ, μ). In the past it was not possible to calculate matrix elements between different SU(3) irreps (except for a few relatively simple cases), but recently a general code was released that removes this limitation [14]. The point is that the terms that are part of the traditional Hamiltonian of Eq.(1) do not couple different SU(3) irreps while the pairing interaction has non-vanishing matrix elements between states of different (λ, μ). It follows from this that the calculation is far more interesting and challenging when the pairing is present [15].

The paper is organized as follows: In the next section the structure of basis functions and operators of the generalized pseudo-SU(3) model are defined and general expressions for evaluating one-body and two-body matrix elements are determined. In Section 3, the pairing interaction is expanded in terms of SU(3) tensor operators and expressions for its matrix elements are given. The importance of pairing for the pseudo-SU(3) model is illustrated in Section 4 by investigating its influence on wavefunctions and excitation spectra. A summary is presented in Section 5.

2 Wavefunctions, operators, and matrix elements

2.1 Wavefunctions

Wavefunctions of the pseudo-SU(3) model which take the spin and proton-neutron degrees of freedom explicitly into account are labeled by the eigenvalues of the Casimir invariants of the group chains depicted in Fig. 1.

Figure 1

Since the pseudo-SU(3) model is a many-particle theory, it is to be understood that its

wavefunctions are subject to the Pauli Exclusion Principle, that is, they must be anti-symmetric under the interchange of identical particles. Because a particle-permutation $\leftrightarrow$ unitary-symmetry complementarity applies, this requirement can be met by restricting the basis states to the antisymmetric irreps of $U(2\Omega_\sigma)$, where $\sigma = \pi$ for protons and $\sigma = \nu$ for neutrons [16] and where $\Omega_\sigma = (\eta_\sigma + 1)(\eta_\sigma + 2)/2$ is the number of normal parity levels in the η_σ shell.

These anti-symmetric irreps of $U(2\Omega_\sigma)$ are characterized by $\left[1^{N_\sigma}\right]$ where N_σ is the number of normal parity particles. The N_σ in this result can be determined by a simple procedure with the help of a Nilsson diagram (see, for example, [17]): At the experimentally determined β_2 value, the proton (neutron) Nilsson levels (unique parity as well as normal parity) are filled pairwise (to accomodate the spin degree of freedom) from the bottom of the well up until the total number of protons (neutrons) equals the number in the nucleus under consideration. If shifts in β_2 by 10 to 20 percent of the original value do not influence the distribution of particles in the unique and normal parity orbitals, one can assume that the N_σ determined in this way is more or less unique for that particular nucleus. (A Nilsson-like single-particle code which improves upon and automates this procedure is currently being developed. Specifically, for any given nucleus it calculates N_σ as a function of the deformation β_2 together with statistical measures that characterize the sensitivity of the N_σ to changes in β_2.)

The irreps of $U(\Omega_\sigma)$ and $U(2)^{(\sigma)}$ are denoted by $[f_\sigma]$ and $[\bar{f}_\sigma]$, respectively. The overall antisymmetry in $U(2\Omega_\sigma)$ and the particle-permutation $\leftrightarrow$ unitary-symmetry complementarity means that $[f_\sigma]$ is related to $[\bar{f}_\sigma]$ through row $\leftrightarrow$ column exchange [16]. Since the overall attractive nature of the nucleon- nucleon interaction insures that spatially symmetric irreps of the $U(\Omega_\sigma)$ lie energetically lowest, in this work only the most symmetric irreps of $U(\Omega_\pi)$ and $U(\Omega_\nu)$ are taken into account. The physical consequence of this restriction is that no couplings to spin-flip excitations are considered. This approximation can be justified by noting that spin- flip modes normally lie significantly higher in energy than the low-energy collective excitations which are of primary interest here.

The reduction $U(\Omega_\sigma) \supset SU(3)$ (see Fig. 1) determines the $(\lambda_\sigma, \mu_\sigma)$ irreps that are contained in a given $[f_\sigma]$. Multiple occurences of the same $(\lambda_\sigma, \mu_\sigma)$ in a fixed $[f_\sigma]$ are distinguished by a running integer index α_σ. The SU(3) labels of the total wavefunction are calculated by taking all possible products $\{(\lambda_\pi, \mu_\pi) \times (\lambda_\nu, \mu_\nu)\} \rightarrow (\lambda, \mu)$ into account and numbering multiplicities in this product by a running integer index ρ. The total orbital angular momentum L is determined via the chain $SU(3) \supset SO(3)$ where κ labels its multiplicity. Likewise, the total spin of the protons, S_π, and neutrons, S_ν, are coupled to the total spin of the nucleus using the usual rules for coupling angular momentum. And finally, the total angular momentum J results from the coupling of total orbital angular momentum L with the total spin S.

These group structures, reductions, and couplings are an integral part of the pseudo-SU(3) model and all of this information enters the corresponding wave functions:

$$| \left\{ N_\pi[f_\pi] \alpha_\pi \left(\lambda_\pi,\, \mu_\pi\right), N_\nu[f_\nu] \alpha_\nu \left(\lambda_\nu,\, \mu_\nu\right) \right\} \rho \left(\lambda,\, \mu\right) \kappa L \, \left\{ S_\pi, S_\nu \right\} \, S \, ; JM \rangle$$

$$
\begin{aligned}
&= \sum_{\{-\}} \langle\, (\lambda_\pi,\, \mu_\pi)\, \kappa_\pi L_\pi M_{L_\pi}\, ; (\lambda_\nu,\, \mu_\nu)\, \kappa_\nu L_\nu M_{L_\nu} \,|\, (\lambda,\, \mu)\, \kappa L M_L \,\rangle_\rho \\
&\quad \times \langle\, S_\pi M_{S_\pi},\, S_\nu M_{S_\nu} \,|\, S M_S \rangle \;\times\; \langle\, L M_L,\, S M_S \,|\, J M \rangle \\
&\quad \times |N_\pi[f_\pi] \alpha_\pi\, (\lambda_\pi,\, \mu_\pi)\, \kappa_\pi L_\pi M_{L_\pi}\, S_\pi M_{S_\pi} \rangle \;\times\; |N_\nu[f_\nu] \alpha_\nu\, (\lambda_\nu,\, \mu_\nu)\, \kappa_\nu L_\nu M_{L_\nu}\, S_\nu M_{S_\nu} \rangle \\
&= \sum_{\{-\}} \langle\, (\lambda_\pi,\, \mu_\pi)\, \kappa_\pi L_\pi\, ; (\lambda_\nu,\, \mu_\nu)\, \kappa_\nu L_\nu \,||\, (\lambda,\, \mu)\, \kappa L \,\rangle_\rho \;\times\; \langle\, L_\pi M_{L_\pi},\, L_\nu M_{L_\nu} \,|\, L M_L \rangle \\
&\quad \times \langle\, S_\pi M_{S_\pi},\, S_\nu M_{S_\nu} \,|\, S M_S \rangle \;\times\; \langle\, L M_L,\, S M_S \,|\, J M \rangle \\
&\quad \times |N_\pi[f_\pi] \alpha_\pi\, (\lambda_\pi,\, \mu_\pi)\, \kappa_\pi L_\pi M_{L_\pi}\, S_\pi M_{S_\pi} \rangle \;\times\; |N_\nu[f_\nu] \alpha_\nu\, (\lambda_\nu,\, \mu_\nu)\, \kappa_\nu L_\nu M_{L_\nu}\, S_\nu M_{S_\nu} \rangle
\end{aligned}
\tag{2}
$$

with the abbreviation $\{-\} = \{M_{S_\pi},\, M_{S_\nu},\, M_S,\, M_{L_\pi},\, M_{L_\nu},\, M_L,\, \kappa_\pi,\, \kappa_\nu,\, L_\pi,\, L_\nu\}$. In this result the $\langle\, L_1 M_{L_1},\, L_2 M_{L_2} \,|\, L M_L \rangle$ factors are SU(2) Clebsch-Gordan coefficients and the SU(3) coupling coefficients $\langle \ldots ; \ldots | \ldots \rangle$ and $\langle \ldots ; \ldots || \ldots \rangle$ are defined in [18]. The notation of Eq.(2) indicates the following couplings:

- SU(2): $[L_\pi \times L_\nu]^{L M_L}$, $[S_\pi \times S_\nu]^{S M_S}$, and $[L \times S]^{J M}$ where $[A \times B]^{C M_C}$ denotes the usual angular momentum coupling [19, 20];
- SU(3): $\{(\lambda_\pi,\, \mu_\pi) \times (\lambda_\nu,\, \mu_\nu)\} \;\rightarrow\; \{\rho\, (\lambda, \mu)\, \kappa\, L\, M\}$ where the multiplicity index ρ numbers the multiple occurences of (λ, μ) in the product $\{(\lambda_\pi,\, \mu_\pi) \times (\lambda_\nu,\, \mu_\nu)\}$.

2.2 Operators and their matrix elements

As explained in the introduction, accepting the fact that SU(3) is a good pseudo-space symmetry is a fundamental assumption of the pseudo-SU(3) model. The validity of this approximation, which applies to nuclei with $A \geq 100$, is based on:

- the goodness of pseudo-spin symmetry, which is suggested by the Nilsson model and confirmed by other more sophisticated theoretical analyses [3, 4, 7];
- the observation that most heavy nuclei are deformed, which is important because the goodness of the SU(3) symmetry increases with increasing deformation.

And as shown in Section 2, there is a group-subgroup chain for labeling basis states that has SU(3) as an integral part of its structure, and which therefore takes full advantage of the pseudo-SU(3) model symmetry. To guarantee wavefunction-operator compatibility, it is also necessary to express operators associated with physical observables in terms of SU(3) tensors. The procedure for doing this can be found in reference [10]. In this contribution, the spin and proton-neutron degrees of freedom are explicitly incorporated into that formalism. For the sake of completeness, it is useful to repeat some of the

material given in reference [10] as it forms a backdrop for the introduction of an extended notation that allows pairing correlations to be incorporated into the pseudo-SU(3) model.

The formalism requires one to know the relevant group's coupling coefficients and the tensorial properties of the single-particle creation and annihilation operators, which for a fermion system obey the usual anticommutation relations:

$$\{a_i^+,\, a_k\}_+ = \delta_{ik},\;\; \{a_i^+,\, a_k^+\}_+ = \{a_i,\, a_k\}_+ = 0\,, \tag{3}$$

where i (and k) denote the full set of single- particle quantum numbers $\left\{(\eta,0)\, lm\, \frac{1}{2} m_s\right\}$. Here $(\eta,0)$ are the single-particle SU(3) labels $(\lambda,\, \mu)$ and l, $\frac{1}{2}$, m and m_s stand, respectively, for the single- particle orbital angular momentum, spin, and their z-projections. The effect of a creation operator on the single-particle vacuum is given by

$$a^+_{(\eta,0)\, lm\, \frac{1}{2} m_s}\, |0\rangle = |\, (\eta,0)\, lm\, \frac{1}{2} m_s\rangle\,. \tag{4}$$

While the creation operator $a^+_{(\eta,0)\, lm\, \frac{1}{2} m_s}$ is a proper SU(3) tensor, the annihilation operator is not and must be modified appropriately to achieve this status [3, 10, 21]:

$$\tilde{a}_{(0,\eta)\, lm\;\, \frac{1}{2} m_s} \equiv (-1)^{\eta+l+m+\frac{1}{2}+m_s} a_{(\eta,0)\, l-m\, \frac{1}{2}-m_s}\,. \tag{5}$$

With these elementary definitions in place, it is possible to construct three distinguishable tensor operators as products of two creation and/or annihilation operators:

- A one-body (unit tensor) operator,

$$\begin{aligned}
{}^{(1,1)}\mathcal{F}^{(\lambda,\mu)\,\kappa L M_L\, S M_S}_{(\eta',0)(0,\eta)\frac{1}{2}\frac{1}{2}} &\equiv \left\{a^+_{(\eta',0)\,\frac{1}{2}} \times \tilde{a}_{(0,\eta)\,\frac{1}{2}}\right\}^{(\lambda,\mu)\,\kappa L M_L\, S M_S} \\
&= \sum_{\{-\}} \langle\, (\eta',0)\, 1 l'\, (0,\eta)\, 1 l || (\lambda,\, \mu)\, \kappa L\rangle \;\times \langle\, l' m',\, l m\,|\, L M_L\rangle \\
&\quad \times \langle\, \frac{1}{2} m'_s,\, \frac{1}{2} m_s\,|\, S M_S\rangle\; a^+_{(\eta',0)\, l' m'\, \frac{1}{2} m'_s}\; \tilde{a}_{(0,\eta)\, lm\, \frac{1}{2} m_s}\,,
\end{aligned} \tag{6}$$

  with the abbreviation $\{-\} = \{l,\, m,\, m_s,\, l',\, m',\, m'_s\}$ and where the left superscript denotes respectively the number of creation and annihilation operators in the product. Note that the multiplicity index ρ connected with the generic SU(3) product $\{(\lambda_1,\, \mu_1) \times (\lambda_2,\, \mu_2)\} \to \rho\,(\lambda,\mu)$ is usually dropped when the coupling is mutliplicity free, that is, whenever $\rho_{\max} = 1$. To summarize, the couplings in Eq.(6) are:

  - SU(2): $[l' \times l]^{L M_L}$, $[\frac{1}{2} \times \frac{1}{2}]^{S M_S}$;
  - SU(3): $\{(\eta',\, 0) \times (0,\, \eta)\} \to \{\rho_{\max} = 1\,, (\lambda,\mu)\}$.

- An operator that creates a pair of particles,

$$ {}^{(2,0)}\mathcal{F}^{(\lambda,\mu)\,\kappa L M_L\, S M_S}_{(\eta',0)(\eta,0)\frac{1}{2}\frac{1}{2}} \equiv \left\{a^+_{(\eta',0)\,\frac{1}{2}} \times a^+_{(\eta,0)\,\frac{1}{2}}\right\}^{(\lambda,\mu)\,\kappa L M_L\, S M_S}\,,$$

where the coefficients that enter to effect the SU(2) and SU(3) couplings in this case are similar to those that appear in Eq.(6).

- A pair annihilation operator,

$$ {}^{(0,2)}\mathcal{F}^{(\mu,\,\lambda)\,\kappa L M_L\,SM_S}_{(0,\eta')(0,\eta)\frac{1}{2}\frac{1}{2}} \;\equiv\; \left\{\tilde{a}_{(0,\eta')\,\frac{1}{2}} \times \tilde{a}_{(0,\eta)\,\frac{1}{2}}\right\}^{(\mu,\,\lambda)\,\kappa L M_L\,SM_S} . $$

In a completely analogous manner, it is possible to define two-body SU(3) unit tensors as coupled products of the ${}^{(2,0)}\mathcal{F}$ and ${}^{(0,2)}\mathcal{F}$ operators:

$$ {}^{(2,2)}\mathcal{F}^{\rho\,(\lambda,\,\mu)\,\kappa L M_L\,SM_S}_{(\lambda_1,\,\mu_1)(\mu_2,\,\lambda_2)\,S_1\,S_2} \;\equiv\; \left\{ {}^{(2,0)}\mathcal{F}^{(\lambda_1,\,\mu_1)\,S_1}_{(\eta_1',0)(\eta_1,0)\frac{1}{2}\frac{1}{2}} \;\times\; {}^{(0,2)}\mathcal{F}^{(\mu_2,\,\lambda_2)\,S_2}_{(0,\eta_2')(0,\eta_2)\frac{1}{2}\frac{1}{2}} \right\}^{\rho\,(\lambda,\,\mu)\,\kappa L M_L\,SM_S} . \quad (7) $$

Note, that the indices η_1', η_1, η_2', η_2 are suppressed in ${}^{(2,2)}\mathcal{F}$. This simplifies the notation and can be done because these quantum numbers enter into a pseudo-SU(3) model calculation as fixed parameters. In fact, the constraints $\eta_1' = \eta_1$, $\eta_2' = \eta_2$, and $\eta_1 = \eta_2$ apply since inter-shell couplings are not part of a $0\hbar\omega$ theory. These restrictions are lifted in a pseudo-symplectic extension of the pseudo-SU(3) model [4], and therefore for such applications the notation must be expanded appropriately.[1]

The definitions of ${}^{(1,1)}\mathcal{F}$ and ${}^{(2,2)}\mathcal{F}$ as one-body and two-body operators, respectively, are applicable to identical particle systems. The next logical step is the introduction of proton- neutron SU(3)-coupled operators. For example, the simplest multipole-multipole proton- neutron tensor operator can be defined as follows:

$$ \begin{aligned} {}^{\pi\nu}\mathcal{F}^{\rho_o\,(\lambda_o,\,\mu_o)\,\kappa_o L_o S_o;\,J_o M_o}_{(\lambda_{\pi_o},\,\mu_{\pi_o})(\lambda_{\nu_o},\,\mu_{\nu_o})\,S_{\pi_o}\,S_{\nu_o}} \;&\equiv\; \left\{ \mathcal{F}^{(\lambda_{\pi_o},\,\mu_{\pi_o})\,S_{\pi_o}}_{(\eta_{\pi_o}',0)(0,\eta_{\pi_o})\frac{1}{2}\frac{1}{2}} \times \mathcal{F}^{(\lambda_{\nu_o},\,\mu_{\nu_o})\,S_{\nu_o}}_{(\eta_{\nu_o}',0)(0,\eta_{\nu_o})\frac{1}{2}\frac{1}{2}} \right\}^{\rho_o\,(\lambda_o,\,\mu_o)\,\kappa_o L_o S_o;\,J_o M_o} \\ &= \sum_{M_{L_o},M_{S_o}} \langle\, L_o M_{L_o},\; S_o M_{S_o} \,|\, J_o M_o \rangle \\ &\quad \times \left\{ \mathcal{F}^{(\lambda_{\pi_o},\,\mu_{\pi_o})\,S_{\pi_o}}_{(\eta_{\pi_o}',0)(0,\eta_{\pi_o})\frac{1}{2}\frac{1}{2}} \times \mathcal{F}^{(\lambda_{\nu_o},\,\mu_{\nu_o})\,S_{\nu_o}}_{(\eta_{\nu_o}',0)(0,\eta_{\nu_o})\frac{1}{2}\frac{1}{2}} \right\}^{\rho\,(\lambda_o,\,\mu_o)\,\kappa_o L_o M_{L_o} S_o M_{S_o}} . \end{aligned} \quad (8) $$

The subscript "o" in this expression refers to "operator" and is introduced in view of the forthcoming calculation of matrix elements in the pseudo-SU(3) model. The left superscript $\pi\nu$ on the ${}^{\pi\nu}\mathcal{F}$ operator, which is a shorthand notation for the complete label

[1] A complete theory would also include a general discussion of non-particle number conserving as well as particle number conserving operators in both normal-ordered (creation operators to the left of annihilation operators) and multipole (for example, products of one-body tensors) forms. Indeed, it should be clear that an n-body operator (${}^{(n,n)}\mathcal{F}$) can always be expanded in multipole form (and vice-versa, of course) by using the fundamental commutation relations, Eq.(3), together with recoupling formulae to appropriately rearrange (group/order) the creation and annihilation operators (see [22]). Non-particle number conserving operators (${}^{(m,n)}\mathcal{F}$ with $m \neq n$), represent pickup (stripping) type phenomena when $m < n$ ($m > n$). Here it is enough to limit the development to the forms already introduced as these suffice for our Hamiltonian, inclusive of the pairing interaction.

$\{(m_\pi, n_\pi)(m_\nu, n_\nu)\} = \{(1,1)(1,1)\}$, denotes the fact that this operator acts simultaneously in the proton and neutron spaces. The left superscript on each of the $\mathcal{F}$ factors in Eq.(8) is suppressed because it is (1,1) in both cases, that is, $\mathcal{F} = {}^{(1,1)}\mathcal{F}$.

In general a more explicit and complete notation is required:

$$
\begin{aligned}
&{}^{\pi\nu}\mathcal{F}^{\rho_o(\lambda_o,\,\mu_o)\,\kappa_o L_o S_o;\,J_o M_o}_{\{\pi\}\,\{\nu\}} \\
&= \left\{ {}^{\pi}\mathcal{F}^{\rho_{\pi_o}\,(\lambda_{\pi_o},\,\mu_{\pi_o})\,S_{\pi_o}}_{\alpha_{\pi_1}(\lambda_{\pi_1},\,\mu_{\pi_1})\alpha_{\pi_2}(\mu_{\pi_2},\,\lambda_{\pi_2})\,S_{\pi_1}S_{\pi_2}} \times {}^{\nu}\mathcal{F}^{\rho_{\nu_o}\,(\lambda_{\nu_o},\,\mu_{\nu_o})\,S_{\nu_o}}_{\alpha_{\nu_1}(\lambda_{\nu_1},\,\mu_{\nu_1})\alpha_{\nu_2}(\mu_{\nu_2},\,\lambda_{\nu_2})\,S_{\nu_1}S_{\nu_2}} \right\}^{\rho_o\,(\lambda_o,\,\mu_o)\,\kappa_o L_o S_o;\,J_o M_o},
\end{aligned}
\tag{9}
$$

where $\{\pi\}$ stands for $\{\,[\alpha_{\pi_1}(\lambda_{\pi_1},\,\mu_{\pi_1}),\,S_{\pi_1}] \times [\alpha_{\pi_2}(\mu_{\pi_2},\,\lambda_{\pi_2}),\,S_{\pi_2}] \rightarrow \rho_{\pi_o}\,(\lambda_{\pi_o},\,\mu_{\pi_o}),\,S_{\pi_o}\}$ and similarly for $\{\nu\}$, and the α_π and α_ν labels represent additional quantum numbers that are required for a unique identification of the ${}^{\pi}\mathcal{F}$ and ${}^{\nu}\mathcal{F}$ factors in the product. In this case the left superscripts π, ν, and $\pi\nu$ stand for the sets (m_π, n_π), (m_ν, n_ν), and $\{(m_\pi, n_\pi)(m_\nu, n_\nu)\}$, respectively. The matrix elements of such operators can be calculated if the matrix elements of the factors are known:

$$
\begin{aligned}
&\langle \{N'_\pi[f'_\pi]\alpha'_\pi\,(\lambda'_\pi,\,\mu'_\pi)\,, N'_\nu[f'_\nu]\alpha'_\nu\,(\lambda'_\nu,\,\mu'_\nu)\,\}\;\rho'(\lambda',\,\mu')\,\kappa' L'\;\{S'_\pi, S'_\nu\}\;S';\,J' \\
&\quad ||{}^{\pi\nu}\mathcal{F}^{\rho_o\,(\lambda_o,\,\mu_o)\,\kappa_o L_o S_o;\,J_o}_{\{\pi\}\,\{\nu\}}|| \\
&\quad \{N_\pi[f_\pi]\alpha_\pi\,(\lambda_\pi,\,\mu_\pi)\,, N_\nu[f_\nu]\alpha_\nu\,(\lambda_\nu,\,\mu_\nu)\,\}\;\rho\,(\lambda,\,\mu)\,\kappa L\;\{S_\pi, S_\nu\}\;S\,;\,J\rangle \\
&= \chi \begin{Bmatrix} L & L_o & L' \\ S & S_o & S' \\ J & J_o & J' \end{Bmatrix} \chi \begin{Bmatrix} S_\pi & S_{\pi_o} & S'_\pi \\ S_\nu & S_{\nu_o} & S'_\nu \\ S & S_o & S' \end{Bmatrix} \sum_{\bar\rho} \langle\,(\lambda,\,\mu)\,\kappa L\,, (\lambda_o,\,\mu_o)\,\kappa_o L_o\,||\,(\lambda,'\,\mu')\,\kappa' L'\,\rangle_{\bar\rho} \\
&\quad \times \sum_{\rho_\pi \rho_\nu} \begin{Bmatrix} (\lambda_\pi,\,\mu_\pi) & (\lambda_{\pi_o},\,\mu_{\pi_o}) & (\lambda'_\pi,\,\mu'_\pi) & \rho_\pi \\ (\lambda_\nu,\,\mu_\nu) & (\lambda_{\nu_o},\,\mu_{\nu_o}) & (\lambda'_\nu,\,\mu'_\nu) & \rho_\nu \\ (\lambda,\,\mu) & (\lambda_o,\,\mu_o) & (\lambda',\,\mu') & \bar\rho \\ \rho & \rho_o & \rho' & \end{Bmatrix} \\
&\quad \times \langle N'_\pi[f'_\pi]\alpha'_\pi(\lambda'_\pi,\,\mu'_\pi)\,S'_\pi\,|||\,{}^{\pi}\mathcal{F}^{\rho_{\pi_o}\,(\lambda_{\pi_o},\,\mu_{\pi_o})\,S_{\pi_o}}_{\alpha_{\pi_1}(\lambda_{\pi_1},\,\mu_{\pi_1})\alpha_{\pi_2}(\mu_{\pi_2},\,\lambda_{\pi_2})\,S_{\pi_1}S_{\pi_2}}\,|||\,N_\pi[f_\pi]\alpha_\pi(\lambda_\pi,\,\mu_\pi)\,S_\pi\rangle_{\rho_\pi} \\
&\quad \times \langle\,N'_\nu[f'_\nu]\alpha'_\nu(\lambda'_\nu,\,\mu'_\nu)\,S'_\nu\,|||\,{}^{\nu}\mathcal{F}^{\rho_{\nu_o}\,(\lambda_{\nu_o},\,\mu_{\nu_o})\,S_{\nu_o}}_{\alpha_{\nu_1}(\lambda_{\nu_1},\,\mu_{\nu_1})\alpha_{\nu_2}(\mu_{\nu_2},\,\lambda_{\nu_2})\,S_{\nu_1}S_{\nu_2}}\,|||\,N_\nu[f_\nu]\alpha_\nu(\lambda_\nu,\,\mu_\nu)\,S_\nu\,\rangle_{\rho_\nu}\,,
\end{aligned}
\tag{10}
$$

where $\chi\{\ldots\}$ denotes a unitary 9-j or Jahn- Hope symbol [20, 23] and $\{\ldots\}$ its SU(3) extension [24]. The triple-barred matrix elements that enter in Eq.(10), $\langle\ldots|||\ldots|||\ldots\rangle$, are reduced with respect to both SU(3) and SO(3) (see Appendix B), and can be evaluated for operators of physical interest with the code introduced in reference [14]. When

this result is used to evaluate matrix elements of either a pure proton $\{(m_\nu, n_\nu) = (0,0)\}$ or a pure neutron $\{(m_\pi, n_\pi) = (0,0)\}$ operator, it simplifies just as for the corresponding SU(2) case; namely, the occurrence of a zero-body factor with its null SU(3) irrep character means that the 9-(λ,μ) coefficient can be reduced to a 6-(λ,μ) coefficient and the corresponding triple-barred reduced matrix element goes to unity. The pairing interaction is a sum of two such operators: $H_P \rightarrow \{{}^{(2,2)(0,0)}\mathcal{F} + {}^{(0,0)(2,2)}\mathcal{F}\}$.

3 The pairing interaction

As stated in the introduction, the inclusion of a short- range interaction of the pairing type in pseudo-SU(3) calculations has not been tested and seems from other independent evidence to be important. But before exploring the influence pairing has on collective nuclear properties, it is necessary to introduce some additional formalism. In second quantization the pairing Hamiltonian [13] is defined as:

$$H_P = -\frac{G}{4}\sum_{\{-\}} a^{+}_{\eta_1 l_1 \frac{1}{2} j_1 m_1} a^{+}_{\overline{\eta}_1 \overline{l}_1 \frac{\overline{1}}{2} \overline{j}_1 \overline{m}_1} a_{\overline{\eta}_2 \overline{l}_2 \frac{\overline{1}}{2} \overline{j}_2 \overline{m}_2} a_{\eta_2 l_2 \frac{1}{2} j_2 m_2}\,, \tag{11}$$

where $\{-\} = \{\eta_1\,, l_1\,, j_1\,, m_1\,, \eta_2\,, l_2\,, j_2\,, m_2\}$ with $(m_i = -j_i, -j_i+1, \ldots, j_i)$ and $(i = 1,2)$. The bar over quantum numbers in Eq.(11) denotes time-reversal and G is the pairing strength, which is normally taken to be somewhat different for protons and neutrons.

Obviously H_P, as given in Eq.(11), is defined in terms of normal-space quantum numbers. However, since all calculations in the pseudo-SU(3) model are performed in the pseudo-space, it is necessary to transform H_P into its pseudo- space representation. An application of the normal $\rightarrow$ pseudo mapping is described in reference [10] for the general case, and in reference [25] for the special case of pairing. The following is the final identical-particle result, expressed in terms of pseudo-space quantum numbers [15]:

$$\begin{aligned} H_P = -\frac{G}{2}\sum_{\{-\}}(-1)^{l_1+l_2}\left((2l_1+1)(2l_2+1)\right)^{\frac{1}{2}} & \left\langle\,(\eta_1,\,0)\,1\,l_1;\,(\eta_1,\,0)\,1\,l_1||(\lambda_1,\,\mu_1)\,1\,0\right\rangle_1 \\ \times \quad & \left\langle\,(\eta_2,0)\;\,1\,l_2;\,(\eta_2,0)\;\,1\,l_2\,||\,(\lambda_2,\mu_2)\;\,1\,0\,\right\rangle_1 \left\langle\,(\lambda_1,\mu_1)\;\,1\,0;\,(\mu_2,\lambda_2)\;\,1\,0\,||\,(\lambda,\mu)\;\,1\,0\,\right\rangle_\rho \\ \times \quad & {}^{(2,2)}\mathcal{F}^{\rho\,(\lambda,\mu)\,\kappa=1\,L=0\,S=0\,J=0}_{(\lambda_1,\mu_1)(\mu_2,\lambda_2)\,S_1=0\,S_2=0} \end{aligned} \tag{12}$$

where $\{-\} = \{l_1, l_2, \lambda_1, \mu_1, \lambda_2, \mu_2, \rho, \lambda, \mu\}$. (The summation over η_1 and η_2 is dropped because the pseudo-SU(3) model is a $0\hbar\omega$ theory and inter-shell couplings are not allowed.) Since the pairing interaction acts only between like nucleons, the proton (neutron) pairing operator H_P^π (H_P^ν) can be obtained by replacing the quantum numbers in the generic expression, Eq.(12), by their proton (neutron) counterparts. And as already noted, since the proton (neutron) pairing interaction is a SU(3) scalar operator in the neutron (proton) subspace, $(\lambda_\nu,\,\mu_\nu) = (0,0)$ $((\lambda_\pi,\,\mu_\pi) = (0,0))$, the expression for its matrix elements

simplify considerably as compared to those of a general proton-neutron interaction, see Eq.(10). Results for the matrix elements of H_P^π and H_P^ν in a proton-neutron basis are given in Appendix A.

4 Pairing and collective states

Since the primary objective of this study is to assess the importance of pairing correlations in heavy nuclei as revealed within the context of pseudo-SU(3) model, it is of interest to study the influence of the pairing interaction on the wavefunctions themselves. Consider the normal-space Hamiltonian

$$H_{QP} = \frac{\chi}{2}\ {}^rQ^a \cdot {}^rQ^a + G_\pi H_P^\pi + G_\nu H_P^\nu + aK_J^2 + bJ^2 \tag{13}$$

where ${}^rQ^a$ denotes the real quadrupole operator, J the collective angular momentum operator, and K_J^2 stands for a residual interaction that is designed to generate K-band splitting (see [26, 27]). Although the deformation driving ${}^rQ^a \cdot {}^rQ^a$ term is not exactly equal to its pseudo-space image $Q^a \cdot Q^a$, the difference is small. Indeed, within the leading SU(3) irrep the induced corrections to energies and electromagnetic transition probabilities have been shown to be smaller than 1% [16]. Since the exact results for the normal-space $\rightarrow$ pseudo-space transformation are known for H_P^π and H_P^ν (see Eqs. 15, 17), and the K_J^2 and J^2 operators transform as scalars,

$$H_{QP} = \frac{\chi}{2}\ Q^a \cdot Q^a + G_\pi H_P^\pi + G_\nu H_P^\nu + aK_J^2 + bJ^2 \tag{14}$$

for practical purposes. (If not explicitly indicated otherwise, the parameters have the numerical values $\chi = 4.32$ keV, $a = 0.202$ MeV, and $b = 9.26$ keV which are compatible with a realistic set taken from best-fit calculations for heavy deformed nuclei [28]).

The results presented next refer to the relatively simple system of two protons and two neutrons in pseudo-shells $\eta_\pi = 3$ and $\eta_\nu = 4$, respectively. Systems with a larger number of particles have also been studied and were found to display the same qualitative behaviour as the $2\pi 2\nu$ system. The results for these more complicated cases will be published in [28] which focuses on complementary characteristics of the pairing interaction. An advantage of the present study is that concerns associated with truncating the model space can be avoided altogther because the $2\pi 2\nu$ system, though rich in structure, is small enough to be solved without invoking truncation measures.

Before taking a closer look at the quadrupole $\leftrightarrow$ pairing strength relation, it is instructive to consider the spectrum for the case of pure pairing, that is, when $\chi = a = b = 0$ and $G_\pi = G_\nu = 1$ MeV in H_{QP}. The results shown in Fig. 2, (A), provide a check on the calculations since in this case analytic results for the excitation energies can be given, and, (B), illustrate the structure of the pairing spectrum for a proton-neutron system.

Figure 2

Analytical results for the pairing spectrum can be derived in a quasi-spin formalism [12, 15] with identical particle states classified according to their seniority s, which is simply the number of particles not coupled to J=0 pairs. For the combined proton-neutron system, the excitation energy $E_{s_\pi s_\nu}$ is given by the sum

$$E_{s_\pi s_\nu} = -\frac{G}{4} \sum_{\sigma\,=\,\pi,\,\nu} \Big(s_\sigma[s_\sigma - 2(\Omega_\sigma + 1)] + 2N_\sigma(\Omega_\sigma + 1) - N_\sigma^2 \Big) \ ,$$

where s_σ denotes the seniority, N_σ the number of nucleons, and Ω_σ the number of levels in the η_σ shell ($\pi \rightarrow$ proton, $\nu \rightarrow$ neutron). (Recall that $\Omega = (\eta + 1)(\eta + 2)/2$) for the η-th major (normal or pseudo) shell.) Taking $N_\pi = N_\nu = 2$ with $\eta_\pi = 3$ and $\eta_\nu = 4$ one obtains a pairing gap of 10 MeV (15 MeV) for the energy required to break a proton (neutron) pair. If one renormalizes the spectrum with respect to the ground state one obtains the results shown in Fig. 2. The states which are ordered according to increasing angular momentum group into degenerate sets classified by the seniority quantum numbers (s_π, s_ν). The dimension of each set, which is given on the top of each horizontal bar,indicates that only a few states contribute to the low energy spectrum. The result of combining this pure pairing seniority level scheme with a quadrupole dominated rotational spectrum to yield more realistic results is considered next.

To investigate the role pairing plays in a description of collective phenomena, consider the $N_\pi = N_\nu = 2$ system introduced above but now with the pairing and quadrupole interaction strengths both taken to be non-zero. Specifically, the quadrupole strength χ was fixed at a realistic value and the pairing strength was varied from 0 to 0.30 MeV. (For simplicity, the proton and neutron pairing strengths were set equal to one another, $G_\pi = G_\nu \equiv G$.) An estimate for realistic G_π and G_ν values can be obtained from various phenomenological formulas, for example, in [29] one finds the result $G_\pi = \frac{17}{A}$ MeV and $G_\nu = \frac{23}{A}$ MeV. For our example of a rare earth nucleus with 2 protons (neutrons) in the pseudo shell $\eta_\pi = 3$ ($\eta_\nu = 4$), which derives from a normal shell with principal quantum number $\eta_\pi = 4$ ($\eta_\nu = 5$), one has $A = 136$ and hence $G_\pi = 0.125$ MeV ($G_\nu = 0.169$ MeV). (Although this system happens to have the proton and neutron numbers of ${}^{136}_{52}\mathrm{Xe}_{84}$, it is important to remember that the purpose of this investigation is to study the influence of the pairing interaction on excitation spectra and not to model a particular physical system. This exercise should be considered a forerunner for forthcoming attempts at describing and predicting experimental data. Best fit calculations for ${}^{136}_{52}\mathrm{Xe}_{84}$ as well as other rare-earth nuclei will be published elsewhere [28].)

Figure 3, 4, 5

Figures 3, 4, and 5 indicate absolute values of the amplitudes (upper plots) and intensities (lower plots) of the calculated 0_1, 4_1, and 8_1 eigenstates. These values (vertical axis) are plotted as a function of the basis state number (running from left to right) and the pairing strength (increasing from front to rear). The basis states have been sorted as a function of

the eigenvalue of the second order SU(3) Casimir invariant, $C_2 = \lambda^2 + \lambda\mu + \mu^2 + 3(\lambda + \mu)$. Hence, in each figure the most deformed SU(3) irrep is associated with the index 1, which is on the far left, and the least deformed with the highest index on the far right. The pairing strength is indicated on the right, where for simplicity the strengths were set equal to one another, $G_\pi = G_\nu \equiv G$.

For $G = 0$ the quadrupole interaction dominates and the ground state wavefunction is comprised of the most deformed SU(3) irrep only; that is, no mixing occurs, and this result is angular momentum independent. As the pairing strength is increased, contributions from irreps that correspond to less deformed configurations grow, but not all basis states contribute in the same way to each eigenstate. Instead, the results suggest that there may be a distinguishable pattern that applies for each value of the total angular momentum. If this turns out to be the case and can be predicted apriori, it could lead to a prescription for an angular momentum dependent model space truncation. This subject is currently under investigation. Regardless of whether this is true or not, the point to note is that for realisitic values of the pairing strength (0.1 MeV $\leq G \leq$ 0.2 MeV) there is significant basis state mixing for all values of the total angular momenta — the pairing interaction definitely breaks the SU(3) symmetry.

Notice that the contribution of the leading irrep increases with increasing angular momentum L. Two complementary factors contribute to this effect: First of all, within any $U(\Omega)$ representation the number of SU(3) irreps that contribute to a particular L value decreases as L increases. Indeed, if (λ, μ) is the leading SU(3) irrep then the value $L = (\lambda + \mu)$ is unique and the probability of it occuring in the eigenstate with $J = L = (\lambda+\mu)$ is unity (100%), independent of the nature of the interaction. Secondly, as the value of L increases, the probability of finding pairs that couple to angular momentum zero decreases because the formation of a zero-coupled pair subtracts angular momentum from the system. Another way of saying the same thing is that the formation of a pair effectively reduces the system to one with two less particles and unless one is near the mid-shell region, the leading SU(3) irrep of such a system has a lower maximum L value.

The influence of the pairing strength G on the rotational ground state band of the $N_\pi = N_\nu = 2$ example is shown in Fig. 6.

Figure 6

With the same parameters as used in the above analyses, one clearly finds that with G increasing from 0.0MeV to 0.3MeV the system moves from a pure rotational structure towards one that is like that of the pure pairing Hamiltonian, see Fig. 2 for comparison. For realistic values of G (0.1 MeV $\leq G \leq$ 0.2 MeV) there is clearly competition between the quadrupole dominated rotational picture and the pairing dominated seniority scheme.

Effective moments of inertia θ for each of the excited levels in Fig. 6 are shown in Fig. 7. The values plotted were extracted from the spectrum using the simple formula $\theta = \frac{L(L+1)}{2E}$. The fact that the moments of inertia decrease with increasing pairing strength

is to be expected because typical 2^+ states of rotational bands lie at a few tens of keV while those of pairing- dominated vibrator configurations are usually in the MeV range. Note that for a fixed value of the pairing strength the magnitude of the decrease in the moments of inertia is less the higher the L value. Again, this is the expected result because states with high angular momentum as compared to those with low values resist the formation of pairs because each pair subtracts angular momentum from the system, thereby making rotation more difficult to generate. So while from Fig. 6 an increase in the pairing appears to affect states with high L more strongly than those with lower L values, the opposite holds. The former is an illusion created by the $L(L+1)$ multiplier that enters into the energy equation; the pairing interaction is most effective in states with low L values. This picture is consistent, for example, with the interpretation of backbending as being associated with the break-up of a pair.

5 Summary and outlook

The formalism that is required for carrying out shell- model calculations within the framework of the pseudo-SU(3) model, extended to include explicitly both the spin and proton-neutron degrees of freedom, has been introduced. In particular, the pairing interaction, which couples different irreps of SU(3), has been expressed in terms of pseudo-space unit tensor operators, and general expressions for calculating the matrix elements of these tensors, and hence the pairing interaction have been given. (Another study will give a more detailed account of the pairing force and explore additional features that reflect on the importance of pairing correlations in heavy nuclei [15].) The presented results make liberal use of an extended technology for calculating SU(3) coupling and recoupling coefficients [18] and a recently released code for evaluating SU(3) reduced matrix elements [14].

The influence of pairing correlations on the collective (rotational) properties of the $N_\pi = 2$ ($\eta_\pi = 3$) and $N_\nu = 2$ ($\eta_\nu = 4$) system was then considered. The Hamiltonian that was used includes a $Q^a \cdot Q^a$ term, the pairing interaction, and residual J^2 and K_J^2 terms that serve to generate the $J(J+1)$ and K-band splitting. (The splitting generated by J^2 is trivial because rotational invariance insures that J is a good quantum number, but since K_J is not an exact symmetry the splitting generated by K_J^2 is non- trivial, and can compete, for example, with that generated by the pairing interaction.) For vanishing pairing strengths ($G_\pi = G_\nu = 0$), this Hamiltonian has non-vanishing matrix elements between states of the same irrep only. In particular, for $J = 0$ states neither the J^2 and K_J^2 terms contribute so only the $Q^a \cdot Q^a$ term enters and as a consequence the leading SU(3) irrep, which is unique, is the only contributor to the ground state. Upon increasing the pairing strength, however, this perfect SU(3) symmetry is broken and other irreps enter, especially in the ground state. Similar results are observed for the states of higher angular momentum, but to a lesser degree because for these the role of the pairing interaction is curtailed and the K_J^2 term enters.

This paper is the first in a series of contributions which are aimed at achieving a proper

quantitative description of heavy nuclei within the framework of the pseudo-SU(3) model, with pairing playing its proper role. In keeping with this plan, the general formalism for calculating the matrix elements of any interaction was introduced, with explicit results given for the pairing interaction. To gain a deeper understanding of the importance of short-range correlations in strongly deformed systems, it will certainly be useful to investigate the effect pairing has on other collective nuclear properties such as g_R factors, B(M1) values, K_J mixing, B(E2) values, etc. Such studies will serve as the basis for a quantitative description of heavy nuclei within the framework of the pseudo-SU(3) model. Beyond these studies, however, lies an even bigger challenge, namely, the development of a comprehensive program aimed at the unification of shell-model and collective-model theories. It is our goal to be able to give a proper account of macroscopic phenomena in terms of a user-friendly and usable microscopic theory. The simplifications provided by the pseudo-spin concept are, of course, key to achieving this lofty objective.

6 Appendix

A Proton and neutron pairing matrix elements

Applying the general result given in Section 2 for SU(3) matrix elements to the proton pairing operator, see Eq.(12), yields:

$$\langle \{N'_\pi[f'_\pi]\alpha'_\pi\,(\lambda'_\pi,\,\mu'_\pi)\,, N'_\nu[f'_\nu]\alpha'_\nu\,(\lambda'_\nu,\,\mu'_\nu)\,\}\;\rho'(\lambda',\,\mu')\,\kappa' L'\;\{S'_\pi, S'_\nu\}\;S';\,J'$$

$$||H^\pi_P||$$

$$\{N_\pi[f_\pi]\alpha_\pi\,(\lambda_\pi,\,\mu_\pi)\,, N_\nu[f_\nu]\alpha_\nu\,(\lambda_\nu,\,\mu_\nu)\,\}\;\rho\,(\lambda,\,\mu)\,\kappa L\;\{S_\pi, S_\nu\}\;S\,;J\rangle$$

$$= \delta_{LL'}\delta_{SS'}\delta_{JJ'}\delta_{S_\pi S'_\pi}\delta_{S_\nu S'_\nu}\delta_{\alpha_\nu\alpha_{\nu'}}\delta_{\lambda_\nu\lambda_{\nu'}}\delta_{\mu_\nu\mu_{\nu'}}\sqrt{\frac{2J+1}{(2L+1)(2S_\pi+1)(2S_\nu+1)}}$$

$$\times \sum_{\lambda_0,\,\mu_0,\bar\rho} \langle\,(\lambda,\,\mu)\,\kappa L\,,(\lambda_o,\,\mu_o)\,\kappa_o = 1 L_o = 0\,||\,(\lambda',\,\mu')\,\kappa' L'\,\rangle_{\bar\rho}$$

$$\times \sum_{\rho_\pi} \left\{ \begin{array}{cccc} (\lambda_\pi,\,\mu_\pi) & (\lambda_o,\,\mu_o) & (\lambda'_\pi,\,\mu'_\pi) & \rho_\pi \\ (\lambda_\nu,\,\mu_\nu) & (0,\,0) & (\lambda'_\nu,\,\mu'_\nu) & 1 \\ (\lambda,\,\mu) & (\lambda_o,\,\mu_o) & (\lambda',\,\mu') & \bar\rho \\ \rho & 1 & \rho' & \end{array} \right\}$$

$$\times \sum_{\{-\}} C^{\rho_{12}\,(\lambda_o,\mu_o)}_{(\lambda_1,\mu_1)\,(\mu_2,\lambda_2)} \langle N'_\pi[f'_\pi]\alpha'_\pi(\lambda'_\pi,\,\mu'_\pi), S'_\pi\,|||\mathcal{T}^{\rho_{12}\,(\lambda_o,\mu_o)\,S_{\pi_o}=0}_{(\lambda_1,\mu_1)\,(\mu_2,\lambda_2)}|||\,N_\pi[f_\pi]\alpha_\pi(\lambda_\pi,\,\mu_\pi), S_\pi\rangle_{\rho_\pi} \tag{15}$$

where $\{-\} = \{\lambda_1,\,\mu_1,\,\lambda_2,\,\mu_2,\,\rho_{12}\}$. The triple barred reduced matrix element that enters here is defined in Appendix B. The expansion coefficients $C^{\rho_{12}\,(\lambda_o,\mu_o)}_{(\lambda_1,\mu_1)\,(\mu_2,\lambda_2)}$ are given by:

$$\begin{aligned} C^{\rho_{12}\,(\lambda_o,\mu_o)}_{(\lambda_1,\mu_1)\,(\lambda_2,\mu_2)} \;\equiv\; & \sum_{l_1 l_2} \frac{G}{2}(-1)^{l_1+l_2+1}\left((2l_1+1)(2l_2+1)\right)^{\frac{1}{2}} \\ & \times \;\langle\,(\eta_\pi,\,0)\,1\,l_1;\,(\eta_\pi,\,0)\,1\,l_1||\,(\lambda_1,\,\mu_1)\,1\,L_1 = 0\rangle_1 \\ & \times \;\langle\,(\eta_\pi,0)\;\,1\,l_2;\,(\eta_\pi,0)\;\,1\,l_2\,||\,(\lambda_2,\mu_2)\;\,1\,L_2 = 0\,\rangle_1 \\ & \times \;\langle\,(\lambda_1,\mu_1)\;\,1\,0;\,(\mu_2,\lambda_2)\;\,1\,0\,||\,(\lambda,\mu)\;\,1\,0\,\rangle_{\rho_{12}} \end{aligned} \tag{16}$$

where the constraint $\eta_{\pi_1} = \eta_{\pi_2} \equiv \eta_\pi$ has been invoked because within a pseudo-SU(3) model framework the action of the pairing operator is restricted to one major shell. (See Chapter 7 of [15] for a more complete story, including, among other things, numerical results for the expansion coefficients for the $\eta = 3$ case.)

A complementary result ($\pi \leftrightarrow \nu$, etc.) can be used to calculate matrix elements of the neutron pairing operator:

$$\langle \{ N'_\pi[f'_\pi]\alpha'_\pi\,(\lambda'_\pi,\,\mu'_\pi)\,, N'_\nu[f'_\nu]\alpha'_\nu\,(\lambda'_\nu,\,\mu'_\nu)\,\}\;\rho'(\lambda',\,\mu')\,\kappa' L'\;\{S'_\pi, S'_\nu\}\;S';\;J'$$

$$||H^\nu_P||$$

$$\{ N_\pi[f_\pi]\alpha_\pi\,(\lambda_\pi,\,\mu_\pi)\,, N_\nu[f_\nu]\alpha_\nu\,(\lambda_\nu,\,\mu_\nu)\,\}\;\rho\,(\lambda,\,\mu)\,\kappa L\;\{S_\pi, S_\nu\}\;S\,;J\rangle$$

$$= \delta_{LL'}\delta_{SS'}\delta_{JJ'}\delta_{S_\pi S'_\pi}\delta_{S_\nu S'_\nu}\delta_{\alpha_\pi\alpha_{\pi'}}\delta_{\lambda_\pi\lambda_{\pi'}}\delta_{\mu_\pi\mu_{\pi'}}\sqrt{\frac{2J+1}{(2L+1)(2S_\pi+1)(2S_\nu+1)}}$$

$$\times \sum_{\lambda_0,\,\mu_0,\bar\rho} \langle\,(\lambda,\,\mu)\,\kappa L\,,(\lambda_o,\,\mu_o)\,\kappa_o = 1 L_o = 0\,||\,(\lambda',\,\mu')\,\kappa' L'\,\rangle_{\bar\rho}$$

$$\times \sum_{\rho_\pi} \left\{ \begin{array}{cccc} (\lambda_\pi,\,\mu_\pi) & (0,\,0) & (\lambda'_\pi,\,\mu'_\pi) & 1 \\ (\lambda_\nu,\,\mu_\nu) & (\lambda_o,\,\mu_o) & (\lambda'_\nu,\,\mu'_\nu) & \rho_\nu \\ (\lambda,\,\mu) & (\lambda_o,\,\mu_o) & (\lambda',\,\mu') & \bar\rho \\ \rho & 1 & \rho' & \end{array} \right\}$$

$$\times \sum_{\{-\}} C^{\rho_{12}\,(\lambda_o,\,\mu_o)}_{(\lambda_1,\,\mu_1)\,(\mu_2,\,\lambda_2)} \langle N'_\nu[f'_\nu]\alpha'_\nu(\lambda'_\nu,\,\mu'_\nu)S'_\nu\,|||\mathcal{T}^{\rho_{12}\,(\lambda_o,\,\mu_o)\,S_{\nu_o}=0}_{(\lambda_1,\,\mu_1)\,(\mu_2,\,\lambda_2)}\,|||\,N_\nu[f_\nu]\alpha_\nu(\lambda_\nu,\,\mu_\nu)S_\nu\rangle_{\rho_\nu} \tag{17}$$

with $\{-\} = \{\lambda_1,\,\mu_1,\,\lambda_2,\,\mu_2,\,\rho_{12}\}$. The $C^{\rho_{12}\,(\lambda_o,\,\mu_o)}_{(\lambda_1,\,\mu_1)\,(\mu_2,\,\lambda_2)}$ in this case is for neutrons, that is, $\eta_\pi \rightarrow \eta_\nu$ in Eq.(16).

B Tripple-barred reduced matrix elements

The Wigner-Eckart theorem for SU(2) gives

$$\langle\,(\lambda_3,\,\mu_3)\,\kappa_3 l_3 m_3|\,T^{(\lambda_2,\,\mu_2)\kappa_2 l_2 m_2}\,|\,(\lambda_1,\,\mu_1)\,\kappa_1 l_1 m_1\,\rangle$$

$$= \quad \langle\,l_1 m_1,\,l_2 m_2\,|\,l_3 m_3\,\rangle\,\langle\,(\lambda_3,\,\mu_3)\,\kappa_3 l_3||\,T^{(\lambda_2,\,\mu_2)\,\kappa_2 l_2}\,||\,(\lambda_1,\,\mu_1)\,\kappa_1 l_1\,,\rangle \tag{18}$$

where $\langle\,(\lambda_3,\,\mu_3)\,\kappa_3 l_3||\,T^{(\lambda_2,\,\mu_2)\kappa_2 l_2}\,||\,(\lambda_1,\,\mu_1)\,\kappa_1 l_1\,\rangle$ is a double-barred SU(2)-reduced matrix element. Analogously, it is possible to extend this factorization to SU(3) by introducing SU(3)-reduced triple-barred matrix elements. An important difference in this case, however, is a sum over the multiplicity index ρ:

$$\langle\,(\lambda_3,\,\mu_3)\,\kappa_3 l_3 m_3\,|\,T^{(\lambda_2,\,\mu_2)\kappa_2 l_2 m_2}\,|\,(\lambda_1,\,\mu_1)\,\kappa_1 l_1 m_1\,\rangle$$

$$= \sum_{\rho}\ \langle\,(\lambda_1,\mu_1)\,\kappa_1 l_1 m_1;\,(\lambda_2,\mu_2)\,\kappa_2 l_2 m_2|\,(\lambda\mu)\,\kappa_3 l_3 m_3\rangle_\rho\,\langle\,(\lambda_3,\,\mu_3)\,|||\,T^{(\lambda_2,\,\mu_2)}\,|||\,(\lambda_1,\,\mu_1)\,\rangle_\rho$$

$$= \sum_{\rho}\ \langle\,l_1 m_1,\,l_2 m_2\,|\,l_3 m_3\,\rangle\langle\,(\lambda_1,\mu_1)\,\kappa_1 l_1;\,(\lambda_2,\mu_2)\,\kappa_2 l_2||\,(\lambda_3\mu_3)\,\kappa l_3\rangle_\rho$$

$$\times\langle\,(\lambda_3,\,\mu_3)\,|||\,T^{(\lambda_2,\,\mu_2)}\,|||\,(\lambda_1,\,\mu_1)\,\rangle_\rho\,.$$

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Figure captions

Figure 1: The group chains that are used for the classification of pseudo-SU(3) wavefunctions are depicted in the two center columns. The eigenvalues of the corresponding Casimir invariants, which are indicated on the far left and right of the diagram, provide a complete labeling scheme for pseudo-SU(3) basis states.

Figure 2: The excitation spectrum of H_{QP} for the pseudo- space configuration $[\,(fp)^{N_\pi=2}(gds)^{N_\nu=2}\,]$ for the case of a pure pairing interaction, $\chi = 0$ and $G_\pi = G_\nu = 1$ MeV. The horizontal axis denotes the total angular momentum with $L = J$ since $S = 0$. The seniority quantum numbers, (s_π, s_ν) for protons and neutrons, respectively, are indicated on the far right and the numbers on top of each level bar denote the corresponding dimension.

Figure 3: Spreading of the $L = 0$ ground state over the 29 dimensional $L = 0$ pseudo-SU(3) model basis of the $[\,(fp)^{N_\pi=2}(gds)^{N_\nu=2}\,]$ system. Absolute values of amplitudes (upper graphs) and intensities (lower graphs) are plotted on the vertical axis as a function of the basis state number (sorted according to the eigenvalue of the second order Casimir invariant of SU(3), $C_2 = \lambda^2 + \lambda\mu + \mu^2 + 3(\lambda + \mu)$, and therefore according to their intrinsic deformation, with values decreasing from left to right) along the horizontal axis. The strength G of the pairing interaction is given on the far right, increasing front to rear from 0.0 to 0.3 MeV. For simplicity the proton and neutron pairing strengths were set equal, $G = G_\pi = G_\nu$.

Figure 4: Spreading of the $L = 4$ member of the ground band over the 101 dimensional $L = 4$ pseudo-SU(3) model basis of the $[\,(fp)^{N_\pi=2}(gds)^{N_\nu=2}\,]$ system, see Fig. 3.

Figure 5: Spreading of the $L = 8$ member of the ground band over the 101 dimensional $L = 8$ pseudo-SU(3) model basis of the $[\,(fp)^{N_\pi=2}(gds)^{N_\nu=2}\,]$ system, see Fig. 3.

Figure 6: Influence of the pairing strength (horizontal axis, $G = G_\pi = G_\nu$) on members of the ground state of the $[\,(fp)^{N_\pi=2}(gds)^{N_\nu=2}\,]$ system. Note that the rotational band on the left ($G = 0$) converts into a pairing dominated level structure (seniority level scheme) for ($G = 0.3MeV$) on the right. A realistic range for the pairing strength is approximately (0.1MeV $\leq G \leq$ 0.2 MeV).

Figure 7: Moments of inertia as a function of the pairing strength ($G = G_\pi = G_\nu$) for yrast states of the $[\,(fp)^{N_\pi=2}(gds)^{N_\nu=2}\,]$ system. The moments of inertia θ were extracted from the calculated spectrum, see Fig. 6, by using the schematic formula $\theta = \frac{L(L+1)}{2E}$. Values of the angular momenta ($J = L$ since $S = 0$) are indicated on the right.